Unit 7 Worksheet 1

1. $\cos^3\theta + \sin^2\theta\cos\theta = \cos\theta$

$\cos\theta(\cos^2\theta + \sin^2\theta) =$

$\cos\theta(1) =$

$\cos\theta = \cos\theta$ ✓

2. $\csc^2\theta - \cos^2\theta\csc^2\theta = 1$

$\csc^2\theta(1 - \cos^2\theta) =$

$\csc^2\theta(\sin^2\theta) =$

$\frac{1}{\sin^2\theta}(\sin^2\theta) =$

$1 = 1$

3. $\sec\theta\sin\theta = \tan\theta$

$\frac{1}{\cos\theta}\cdot\sin\theta =$

$\frac{\sin\theta}{\cos\theta} =$

$\tan\theta = \tan\theta$

4. $\frac{\csc\theta}{\sec\theta} = \cot\theta$

$\frac{1}{\sin\theta}\cdot\cos\theta =$

$\frac{\cos\theta}{\sin\theta} =$

$\cot\theta = \cot\theta$

5. $\frac{\sec^2\theta - 1}{\tan\theta} = \tan\theta$

$\frac{\tan^2\theta}{\tan\theta} =$

$\tan\theta = \tan\theta$

6. $\frac{\cot\theta}{\csc^2\theta - 1} = \tan\theta$

$\frac{\cot\theta}{\cot^2\theta} =$

$\frac{1}{\cot\theta} =$

$\tan\theta = \tan\theta$

7. $\sec\theta\sin\theta\cot\theta = 1$

$\frac{1}{\cos\theta}\sin\theta\frac{\cos\theta}{\sin\theta} =$

$1 = 1$

8. $\cot\theta\csc\theta\tan\theta = \sec\theta$

$\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} =$

$\frac{1}{\cos\theta} =$

$\sec\theta = \sec\theta$

9. $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - (1 - \cos^2\theta) =$

$\cos^2\theta - 1 + \cos^2\theta =$

$2\cos^2\theta - 1 = 2\cos^2\theta - 1$

10. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$(1 - \sin^2\theta) - \sin^2\theta =$

$1 - 2\sin^2\theta = 1 - 2\sin^2\theta$

11. $\cot\theta \sin\theta = \cos\theta$

$\frac{\cos\theta}{\sin\theta} \sin\theta =$

$\cos\theta = \cos\theta$

12. $\frac{\tan\theta}{\sec\theta} = \sin\theta$

$\frac{\sin\theta}{\cos\theta} \cdot \cos\theta =$

$\sin\theta = \sin\theta$

$\frac{\sin\theta}{\cos\theta}, \cos\theta$

$\frac{1}{\cos\theta} \cdot \frac{\cos}{1}$

13. $\sin\theta(1+\csc\theta) = \sin\theta + 1$

$\sin\theta + \sin\theta \csc\theta =$

$\sin\theta + \sin\theta \cdot \frac{1}{\sin\theta} =$

$\sin\theta + 1 = \sin\theta + 1$

14. $(1+\tan\theta)^2 = \sec^2\theta + 2\tan\theta$

$1 + 2\tan\theta + \tan^2\theta =$

$(1+\tan^2\theta) + 2\tan\theta =$

$\sec^2\theta + 2\tan\theta = \sec^2\theta + 2\tan\theta$

15. $(1+\tan^2\theta)\cos^2\theta = 1$

$\cos^2\theta + \tan^2\theta \cos^2\theta =$

$\cos^2\theta + \frac{\sin^2\theta}{\cos^2\theta} \cos^2\theta =$

$\cos^2\theta + \sin^2\theta =$

$1 = 1$

16. $\cos\theta = \sec\theta - \sin\theta \tan$

$= \sec\theta - \sin\theta \cdot \frac{\sin}{\cos}$

$= \sec\theta - \frac{\sin^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta}$

$\cos\theta \cdot \cos\theta$

17. $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta) = 1$

$\sec^2\theta - \tan^2\theta =$

$\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} =$

$\frac{1-\sin^2\theta}{\cos^2\theta} =$

$\frac{\cos^2\theta}{\cos^2\theta} =$

$1 = 1$

18. $\frac{\sec\theta}{\csc^2\theta} = \sec\theta - \cos\theta$

$= \frac{1}{\cos\theta} - \cos\theta$

$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$= \frac{1-\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{1}{\csc^2\theta} \cdot \sec\theta$

$\frac{\sec\theta}{\csc^2\theta} \quad \frac{\sec\theta}{\csc^2\theta}$

19. $\frac{1-2\csc\theta}{\cot\theta} = \tan\theta - 2\sec\theta$

$\frac{1}{\cot\theta} - \frac{2\csc\theta}{\cot\theta} =$

$\tan\theta - \frac{2\sin\theta}{\cos\theta \cdot \sin\theta} =$

$\tan\theta - \frac{2}{\cos\theta} =$

$\tan\theta - 2\sec\theta = \tan\theta - 2\sec\theta$

20. $\frac{\sec^2\theta - 1}{\tan\theta} = \tan\theta$

$\frac{\tan^2\theta}{\tan\theta} =$

$\tan\theta = \tan\theta$

21. $\sin\theta + \cos\theta \cot\theta = \csc\theta$

$\sin\theta + \cos\theta \frac{\cos\theta}{\sin\theta} =$

$\left(\frac{\sin^2}{\sin\theta}\right) \sin\theta + \frac{\cos^2\theta}{\sin\theta} =$

$\frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta} =$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta} =$

$\frac{1}{\sin\theta} =$

$\csc\theta = \csc\theta$

22. $\cos\theta(\csc\theta - \sec\theta) = \cot\theta - 1$

$\cos\theta \csc\theta - \cos\theta \sec\theta =$

$\cos\theta \frac{1}{\sin\theta} - \cos\theta \frac{1}{\cos\theta} =$

$\cot\theta - 1 = \cot\theta - 1$

23. $\frac{\cos\theta}{1-\sin^2\theta} = \sec\theta$

$\frac{\cos\theta}{\cos^2\theta} =$

$\frac{1}{\cos\theta} =$

$\sec\theta = \sec\theta$

24. $\tan^2\theta - \tan^2\theta \sin^2\theta = \sin^2\theta$

$\tan^2\theta(1 - \sin^2\theta) =$

$\tan^2\theta \cos^2\theta =$

$\frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta =$

$\sin^2\theta = \sin^2\theta$

25. $\frac{\cot\theta}{1+\cot^2\theta} = \sin\theta\cos\theta$

$\frac{\cot\theta}{\csc^2\theta} =$

$\frac{\cos\theta}{\sin\theta} \cdot \sin^2\theta =$

$\cos\theta\sin\theta =$

$\sin\theta\cos\theta = \sin\theta\cos\theta$

26. $\frac{1+\tan^2\theta}{\cos^2\theta} = \sec^4\theta$

$\frac{\sec^2\theta}{\cos^2\theta} =$

$\sec^2\theta \cdot \sec^2\theta =$

$\sec^4\theta = \sec^4\theta$

27. $\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$

$\frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta} =$

$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} =$

$\sec\theta + \csc\theta = \sec\theta + \csc\theta$

28. $\dfrac{\sec\theta + \tan\theta}{\cos\theta + \cot\theta} = \sin\theta \sec^2\theta$

$$\frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\cos\theta + \frac{\cos\theta}{\sin\theta}} =$$

$$\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta\sin\theta + \cos\theta}{\sin\theta}} =$$

$$\frac{(1+\sin\theta)}{\cos\theta} \cdot \frac{\sin\theta}{(\cos\theta\sin\theta + \cos\theta)} =$$

$$\frac{\sin\theta(1+\sin\theta)}{\cos^2\theta(\sin\theta+1)} =$$

$$\frac{\sin\theta}{\cos^2\theta} =$$

$$\sin\theta \sec^2\theta = \sin\theta \sec^2\theta$$

29. $\dfrac{(1+\sin\theta)^2}{\cos^2\theta} = \dfrac{1+\sin\theta}{1-\sin\theta}$

$$\frac{(1+\sin\theta)^2}{1-\sin^2\theta} =$$

$$\frac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)} =$$

$$\frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\sin\theta}{1-\sin\theta}$$

30. $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta} = \csc\theta$

$$\frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\sin\theta} =$$

$$\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}} =$$

$$\frac{\cos\theta+1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta+\sin\theta\cos\theta} =$$

$$\frac{\cos\theta+1}{\sin\theta+\sin\theta\cos\theta} =$$

$$\frac{(\cancel{\cos\theta}+1)}{\sin\theta(1+\cancel{\cos\theta})} =$$

$$\frac{1}{\sin\theta} =$$

$$\csc\theta = \csc\theta$$

31. $\csc\theta\cos^2\theta + \sin\theta = \csc\theta$

$\csc\theta\cos^2\theta + \sin\theta \cdot \frac{\csc\theta}{\csc\theta} =$

$\csc\theta\left(\cos^2\theta + \sin\theta \frac{1}{\csc\theta}\right) =$

$\csc\theta\left(\cos^2\theta + \sin\theta \cdot \sin\theta\right) =$

$\csc\theta(\cos^2\theta + \sin^2\theta) =$

$\csc\theta(1) =$

$\csc\theta = \csc\theta$

32. $\frac{\csc^2\theta}{\csc^2\theta - 1} = \sec^2\theta$

$\frac{\csc^2\theta}{\cot^2\theta} =$

$\frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta} =$

$\frac{1}{\cos^2\theta} =$

$\sec^2\theta = \sec^2\theta$

33. $\sin\theta\left(\frac{\cot\theta}{\sec\theta} + \csc\theta\right) = \cos^2\theta + 1$

$\sin\theta\left(\frac{\cos\theta}{\sin\theta} \cdot \cos\theta + \frac{1}{\sin\theta}\right) =$

$\sin\theta\left(\frac{\cos^2\theta}{\sin\theta} + \frac{1}{\sin\theta}\right) =$

$\cos^2\theta + 1 = \cos^2\theta + 1$

34. $\frac{2\cos^2\theta - \sin^2\theta + 1}{\cos\theta} = 3\cos\theta$

$\frac{2\cos^2\theta + (-\sin^2\theta + 1)}{\cos\theta} =$

$\frac{2\cos^2\theta + \cos^2\theta}{\cos\theta} =$

$\frac{3\cos^2\theta}{\cos\theta} =$

$3\cos\theta = 3\cos\theta$

35. $\csc\theta - \sin\theta = \cot\theta\cos\theta$

$\frac{1}{\sin\theta} - \sin\theta =$

$\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} =$

$\frac{1-\sin^2\theta}{\sin\theta} =$

$\frac{\cos^2\theta}{\sin\theta} =$

$\frac{\cos\theta}{\sin\theta} \cdot \cos\theta =$

$\cot\theta \cdot \cos\theta = \cot\theta\cos\theta$

36. $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = 2\csc^2\theta$

$\frac{1+\cos\theta + 1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)} =$

$\frac{2}{1-\cos^2\theta} =$

$\frac{2}{\sin^2\theta} =$

$2\csc^2\theta = 2\csc^2\theta$

37. $\frac{1+\tan\theta}{\tan\theta} = 1+\cot\theta$

$\frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta} =$

$\cot\theta + 1 =$

$1+\cot\theta = 1+\cot\theta$

38. $\frac{\cos\theta+\tan\theta}{\sin\theta} = \sec\theta + \cot\theta$

$\frac{\cos\theta}{\sin\theta} + \frac{\tan\theta}{\sin\theta} =$

$\cot\theta + \frac{\sin\theta}{\sin\theta\cos\theta} =$

$\cot\theta + \frac{1}{\cos\theta} =$

$\cot\theta + \sec\theta =$

$\sec\theta + \cot\theta = \sec\theta + \cot\theta$

39. $\csc^4\theta - \cot^4\theta = 2\csc^2\theta - 1$

$\frac{1}{\sin^4\theta} - \frac{\cos^4\theta}{\sin^4\theta} = \frac{2}{\sin^2\theta} - 1$

$\frac{1-\cos^4\theta}{\sin^4\theta} = \frac{2}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\sin^2\theta}$

$\frac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^4\theta} = \frac{2-\sin^2\theta}{\sin^2\theta}$

$\frac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^2\theta\sin^2\theta} =$

$\frac{(1-\cos^2\theta)(1+\cos^2\theta)}{(1-\cos^2\theta)(1+\cos^2\theta)} = \frac{1}{\sin^2\theta} + \cot^2\theta$

$\frac{1+\cos^2\theta}{1-\cos^2\theta}$

$\frac{1+\cos^2\theta}{\sin^2\theta}$

$\csc^2\theta + (\csc^2\theta - 1)$

$\csc^2\theta + \csc^2\theta - 1 =$

$2\csc^2\theta - 1 = \checkmark$

40. $\dfrac{\cos\theta + \cot\theta}{\csc\theta + 1} = \cos\theta$

$$\frac{\cos\theta + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} + 1}$$

$$\frac{\frac{\sin\theta\cos\theta + \cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} + \frac{\sin\theta}{\sin\theta}} =$$

$$\frac{\frac{\sin\theta\cos\theta + \cos\theta}{\sin\theta}}{\frac{1+\sin\theta}{\sin\theta}} =$$

$$\frac{\sin\theta\cos\theta + \cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{1+\sin\theta} =$$

$$\frac{\cos\theta(\sin\theta + 1)}{(1+\sin\theta)} =$$

$$\cos\theta = \cos\theta$$

41. $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta} = \csc\theta$

$$\frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta\cos\theta}{\cos\theta}} =$$

$$\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta(1+\cos\theta)}{\cos\theta}} =$$

$$\frac{\cos\theta+1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta(1+\cos\theta)} =$$

$$\frac{1}{\sin\theta} =$$

$$\csc\theta = \csc\theta$$

42. $\dfrac{2-\sec^2\theta}{\sec\theta} = \dfrac{1-2\sin^2\theta}{\cos\theta}$

$$= \frac{1-2(1-\cos^2\theta)}{\cos\theta}$$

$$= \frac{1-2+2\cos^2\theta}{\cos\theta}$$

$$= \frac{-1+2\cos^2\theta}{\cos\theta}$$

$$= \frac{-1}{\cos\theta}+\frac{2\cos^2\theta}{\cos\theta}$$

$$= -\sec\theta+2\cos\theta$$

$$= -\sec\theta+\frac{2}{\sec\theta}$$

$$= \frac{-\sec^2\theta}{\sec\theta}+\frac{2}{\sec\theta}$$

$$= \frac{2-\sec^2\theta}{\sec\theta}$$

42) $\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$

$\frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} =$

$\frac{1}{\frac{1-\sin\theta}{\cos\theta}}$

$\frac{\cos\theta}{1-\sin\theta} =$

$\frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} =$

$\frac{\cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} =$

$\frac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta} =$

$\frac{\cos\theta}{\cos^2\theta} + \frac{\cos\theta\sin\theta}{\cos^2\theta} =$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} =$

$\sec\theta + \tan\theta = \sec\theta + \tan\theta$

44. $1+\cos\theta = \cot\theta(\sin\theta+\tan\theta)$

$= \cot\theta\sin\theta + \cot\theta\tan\theta$

$= \frac{\cos\theta}{\sin\theta}\sin\theta + 1$

$= \cos\theta + 1$

45. $1-\sin\theta = \tan\theta(\cot\theta - \cos\theta)$

$= \tan\theta\cot\theta - \tan\theta\cos\theta$

$= 1 - \frac{\sin\theta}{\cos\theta}\cdot\cos\theta$

$= 1 - \sin\theta$

46. $\frac{\tan\theta}{1+\tan^2\theta} = \sin\theta\cos\theta$

$\frac{\tan\theta}{\sec^2\theta} =$

$\frac{\frac{\sin\theta}{\cos\theta}}{\sec^2\theta} =$

$\frac{\sin\theta}{\cos\theta\sec^2\theta} =$

$\frac{\sin\theta}{\sec\theta} =$

$\sin\theta\cdot\cos\theta = \sin\theta\cos\theta$

47. $\frac{(\sec\theta-\tan\theta)^2}{\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2} = \frac{1-\sin\theta}{1+\sin\theta}$

$\frac{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2}{\frac{(1-\sin\theta)(1-\sin\theta)}{\cos^2\theta}}$

$\frac{(1-\sin\theta)^2}{1-\sin^2\theta} =$

$\frac{(1-\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)} =$

$\frac{1-\sin\theta}{1+\sin\theta}\quad\frac{1-\sin\theta}{1+\sin\theta}$

48. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\cos\theta}{1-\sin\theta}$

$\sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} =$

$\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} =$

$\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} =$

$\frac{1-\cos\theta}{\sin\theta} =$

49. $\sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}} = \frac{1+\sin\theta}{\cos\theta}$

$\sqrt{\frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}} =$

$\sqrt{\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}} =$

$\sqrt{\frac{1+\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1-\sin\theta}} =$

$\sqrt{\frac{1+\sin\theta}{1-\sin\theta} \cdot \frac{(1+\sin\theta)}{(1+\sin\theta)}} =$

$\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} =$

$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$